Research notes: An approximation model for position reconstruction based on multi-camera views in multi-media setup

Dongning Li, Yuanhui Zhang*, Yan Wei

yzhang1@uiuc.edu

Dept. of Agricultural and Biological Engineering

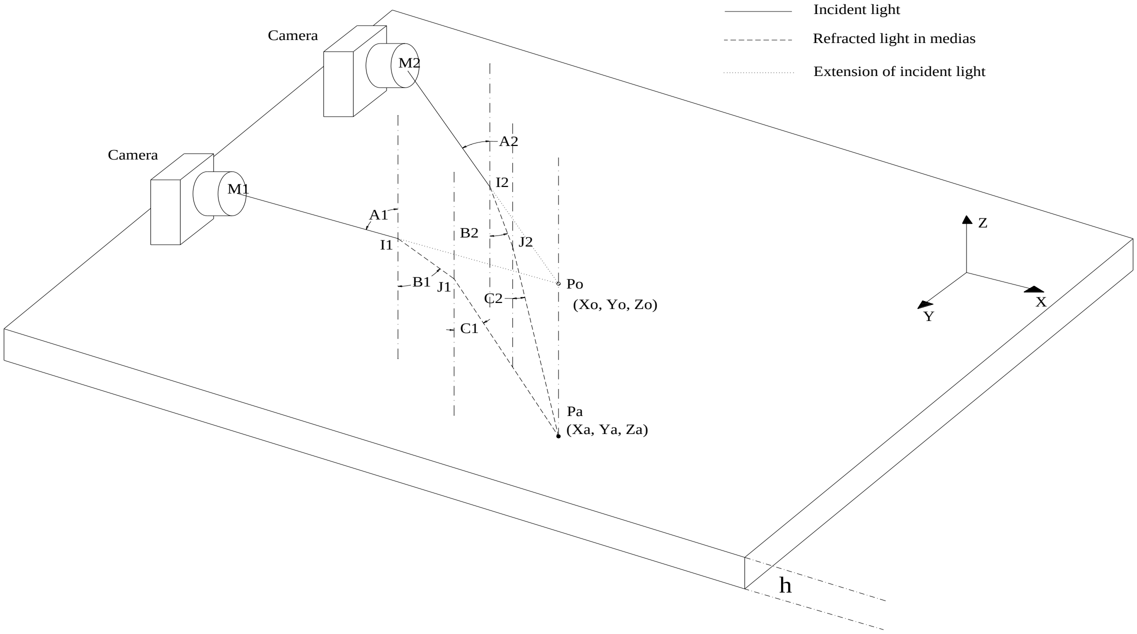
University of Illinois at Urbana Champaign

1 Introduction

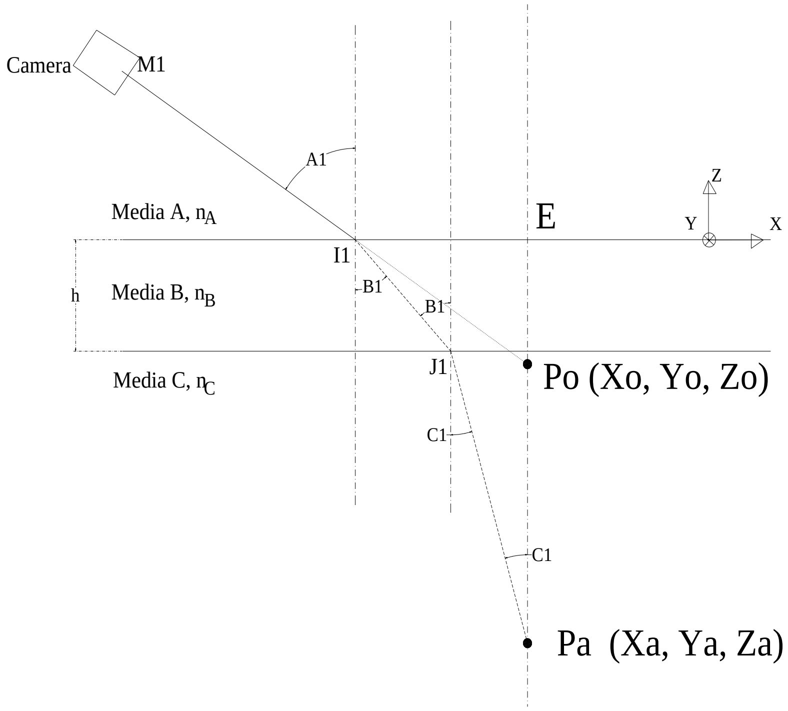
Many times, in particle tracking experiment, the object being studied is a fluid contained in a transparent container. If the cameras are placed in the air, the tracer particles, which are seeded in the fluid, have to be imaged across multi-medias (the fluid, the container wall, and the air). Thus the effect of refraction has to be considered. The ray-tracing model (Maas 1995) for refraction is accurate (contains no systematic error), but unfortunately, complex and time-consuming to implement. In this article, another model is presented. Some approximations are assumed in this model to simplify the math and to make the model easy to be implemented. As the result of the approximation, this model contains a systematic error, which will be small in certain arrangements.

2 Principles

Assume from one particle (P_a), two light rays (P_a-I_1-J_1-M_1, P_a-I_2-J_2-M_2) pass into two cameras through three medias, where P_a is the actual particle position, I_1 J_1, I_2 and J_2 are incident points on the media-changing interfaces, M_1 and M_2 are the points where light rays go into camera lenses. The media-changing interfaces (between media A and B, and between media B and C) are parallel to each other with the distance h (see Figure 1(a)).



(a) Schematic view



(b) View direction parallel to the surface

Figure 1. A particle captured by two cameras with refracted light rays

As described by Snell's Law, when light passes across the interface, it obeys the following rule:

$$\begin{aligned} \sin A_i \cdot n_A &= \sin B_i \cdot n_B \\ \sin B_i \cdot n_B &= \sin C_i \cdot n_C \end{aligned} \tag{1}$$

where A_i stands for the incident angle, B_i stands for the refractive angle (see Figure 1(a), where $i = 1, 2$); n_A, n_B and n_C stand for the refraction indexes of media A, B and C respectively.

Draw a line perpendicular to the interfaces from P_a. Name the intersection of the line and the interface between A and B as E. Name the intersection of M1-I1 and Pa-E as P_o (notice M1-I1 and P_a-E are in the same plane).

In the cases where the cameras are installed so that the lenses are nearly perpendicularly facing toward the media-changing interfaces (the axis of the lens is PERPENDICULAR to the interfaces), the angle A_i B_i and C_i will be small. Thus we have the following APPROXIMATION:

$$\begin{gathered}\frac{\sin A_i}{\sin Bi} \approx \frac{\tan A_i}{\tan B_i} \\ \frac{\sin B_i}{\sin C_i} \approx \frac{\tan B_i}{\tan C_i}\end{gathered} \tag{2}$$

This yields

$$\begin{gathered}\frac{\tan A_i}{\tan B_i} = \frac{n_B}{n_A} \\ \frac{\tan B_i}{\tan C_i} = \frac{n_C}{n_B}\end{gathered} \tag{3}$$

Now let i=1, and we have

$$|P_a E| = \frac{n_C}{n_A}|P_o E| + (1 - \frac{n_C}{n_B})h \tag{4}$$

Notice the length of P_oE ($|P_oE|$) doesn't rely on the incident angle A_1. This means, for any i, the intersection of M_i-I_i and P_a-E is at the same position (P_o). Thus following conclusions can be drawn:

1. Lines M_1-I_1 and M_2-I_2 intercept each other at one point (P_o). This indicates, the images captured by two cameras (in media A) shooting at the object P_a in media C, are identical as the images captured by that two cameras shooting at an object located at P_o in media A. In another word, the refraction forms an image at P_o in media A from the object at P_a in media C.

2. Selecting the following orthogonal coordinate system: X-axis and Y-axis are parallel to the interfaces; Z-axis is perpendicular to the interface. The original point is on the interface between media A and B. The relationship between the P_a's position (X_a, Y_a, Z_a) and the P_o's position (X_o, Y_o, Z_o) will be:

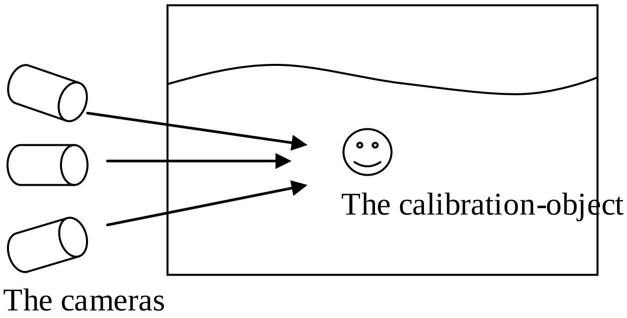
$$
\begin{aligned}
X_a &= X_0 \\
Y_a &= Y_0 \\
Z_a &= \frac{n_C}{n_A} Z_0 + \left(1 - \frac{n_C}{n_B}\right) h
\end{aligned} \tag{5}
$$

Equation (5) gives the transformation between the observed particle position and the actual particle position. Thus, to get the actual particle position, the triangulation method (Malik et al. 1993) can be applied first without considering the refraction issue to get the "observed" particle position P_o, and equation (5) is applied afterwards to get the actual particle position.

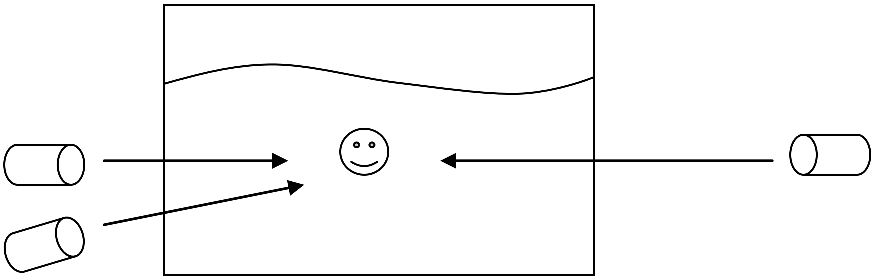
3 Application with Svoboda's toolbox

Assume we want to calibrate several cameras (installed in media A) by Dr. Svoboda's calibration toolbox, according to the toolbox manual, what we do is to use the cameras to capture a moving object in media A and supplies the images to the toolbox.

Now assume the calibration object is immersed in the water contained in a glass tank. If all cameras are seeing the object through the same boundary of the tank, or if all cameras are seeing the object through the boundaries parallel to each other (Figure 2), according to the conclusions from the section 2, the cameras can still be calibrated by directly supplying the images to the toolbox WITHOUT considering the refraction effect. Here instead of the actual-object (at P_a in media C), the object served for the calibration process is the image-object (at P_o in media A) caused by the refraction.



(a) Cameras seeing the object through the same boundary of tank



(b) Cameras seeing the object through the boundaries parallel to each other

Figure 2. Cameras seeing a calibration-object

4 Numerical validation

Data set 371 from the PIV-STD library (Okamoto et al. 2000) was chosen to numerically validate the proposed model. The data can be accessed at:

http://piv.vsj.or.jp/piv/image3d/image371.html

The data set contains the particle-image positions of three cameras viewing the same group of particles. The cameras are installed in air. The particles are seeded in water (with refraction index of 1.33). A zero-thickness glass-wall is vertically installed between the air and the water.

The particle-image coordinates from frame 1 to 10 (in the file ‘ptc001.dat’ to ‘ptc010.dat’) were adopted as the input for Svoboda’s toolbox (see ‘gocal.m’) (Svoboda et al. 2005). The particle-ID provided from the library provides the image-correspondence between the particle-images. The input file ‘points.dat’ and ‘IdMat.dat’ are generated from the ‘ptc*.dat’ files.

The calibration data-set (in ‘ptc999.dat’) which contains 27 uniformly distributed dot-objects in the water was used as the benchmark to validate the calibrated camera-parameters and the proposed model. The image-coordinates of these objects and the calibrated camera-parameters were used to solve the observed spatial-positions of the objects by Svoboda’s toolbox (see ‘gorec.m’). The refraction-compensation method proposed above was then performed to acquire the actual position of the objects.

The normalized position acquired from the above mentioned process is shown in Table 1. The normalized actual position provided from the library is shown in Table 2. It can be found the proposed method has a relative error less than 5%.

The incident angles in this case are about 30 degree. In practice, an incident angle about 10-20 degree should satisfy the approximation and make the method work.

Table 1. Normalized position calculated from the particle-image positions

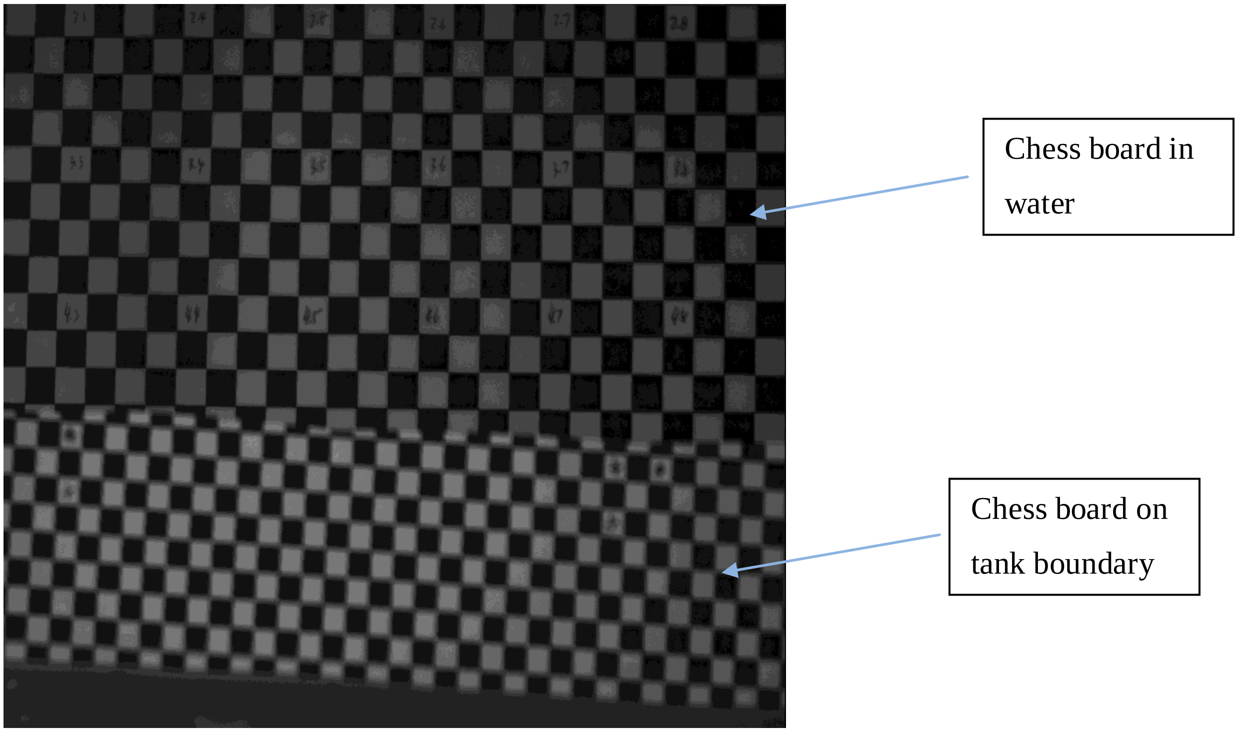
Particle-ID	1	2	3	4	5	6	7	8	9
x	0.9530	0.9877	1.0228	-0.0404	-0.0045	0.0318	-1.0338	-0.9965	-0.9590
y	0.9716	0.9911	1.0105	0.9711	0.9907	1.0102	0.9707	0.9903	1.0099
z	1.0123	-0.0023	-1.0110	1.0119	-0.0011	-1.0080	1.0085	-0.0023	-1.0073
Particle-ID	10	11	12	13	14	15	16	17	18
x	0.9562	0.9909	1.0261	-0.0361	0.0000	0.0363	-1.0281	-0.9909	-0.9533
y	-0.0214	0.0000	0.0214	-0.0215	0.0000	0.0215	-0.0215	0.0000	0.0215
z	1.0119	-0.0011	-1.0085	1.0114	0.0000	-1.0054	1.0081	-0.0012	-1.0047
Particle-ID	19	20	21	22	23	2	25	26	27
x	0.9593	0.9941	1.0292	-0.0317	0.0043	0.0407	-1.0225	-0.9852	-0.9476
y	-1.0117	-0.9885	-0.9652	-1.0115	-0.9882	-0.9648	-1.0111	-0.9877	-0.9643
z	1.0088	-0.0022	-1.0077	1.0084	-0.0010	-1.0046	1.0052	-0.0022	-1.0039

Table 2. The normalized actual positions

Particle-ID	1	2	3	4	5	6	7	8	9
x	1	1	1	0	0	0	-1	-1	-1
y	1	1	1	1	1	1	1	1	1
z	1	0	-1	1	0	-1	1	0	-1
Particle-ID	10	11	12	13	14	15	16	17	18
x	1	1	1	0	0	0	-1	-1	-1
y	0	0	0	0	0	0	0	0	0
z	1	0	-1	1	0	-1	1	0	-1
Particle-ID	19	20	21	22	23	24	25	26	27
x	1	1	1	0	0	0	-1	-1	-1
y	-1	-1	-1	-1	-1	-1	-1	-1	-1
z	1	0	-1	1	0	-1	1	0	-1

5 Really world application

In our study, a three-camera system was calibrated by Svoboda's toolbox as described in section 3. Since it might not be that convenient to move a dot object in water, we used a chess board as the calibration object. The coordinate of the grids were measured by Bouguet's calibration toolbox (http://www.vision.caltech.edu/bouguetj/calib_doc/) and then used to generate the input file ('points.dat', 'IdMat.dat') for Svoboda's toolbox.



Chess board in water

Chess board on tank boundary

Figure 3. Chess board serves as a calibration object

Another chess board (actually, a chess board printed on a paper) was stuck on the outer boundary of the glass tank in order to measure the position and orientation of the tank boundary (also by Svoboda’s toolbox), which is used to establish the coordinate described in equation (5).

6 A trick for PTV application

In PTV application, to regenerate the 3D position of the particles from their images, first the image-correspondence (which image from one camera and which image from another camera are the images of the same particle) has to be established. The positions are calculated afterward by the triangulation method.

It has to be emphasized; the model proposed in section 2 CONTAINS a systematic error due to the approximation described in equation (2), which results in an error in the calibrated camera-parameters. As discussed in section 4, if the image-correspondence is correctly established, the particle positions will be calculated with a moderate error.

However, if the image-correspondence is not correctly established due to the inaccurate camera-parameters, the calculated position will be just non-trustable.

Fortunately, to establish the image-correspondence between two cameras, we don't have to know the full set the camera parameters of both cameras. We only need to know the transformation between the image-coordinate on one camera's image-plane to the epipolar-lines on another camera's image-plane (Maas et al. 1993). This transformation is described by 'fundamental matrix' (see http://en.wikipedia.org/wiki/Fundamental_matrix_(computer_vision)). The fundamental matrix can be solved by the so called "8-point algorithm" (Hartley 1995) when lens distortion can be omitted, or other algorithms (Stein 1997) when lens distortion cannot be omitted. The algorithms to calculate the fundamental matrix are much simpler than the calibration algorithms which calculate the full set of camera parameters, and also yield more accurate result than the calibration algorithms. Here the "accuracy" is measured in terms of the distance between the image (of particle P on camera A's image-plane) and the epipolar-line (which is on camera A's image-plane and is established from the image of particle P on camera B's image-plane).

Thus we would suggest, after the full set of camera parameters have be acquired by a calibration, the fundamental matrix should also be calculated from the images of a calibration-object placed in media C. In PTV data processing routine, first establish the image-correspondence by the fundamental matrixes between each two cameras; employ the calibrated camera-parameters afterward to calculate the position of the particles.

References

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